
Math 4650 - Homework # 3

Infinite Series

Part 1 - Computations

1. Calculate the value of $\sum_{n=2}^{\infty} \left(\frac{2}{7}\right)^n$
 2. Calculate the value of $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{2n}$
 3. Use partial fractions to show that $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} = \frac{1}{2}$
 4. Use partial fractions to show that $\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \frac{3}{2}$
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Part 2 - Proofs

5. Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent series with $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$. Let α be a real number.
 - (a) Prove that $\sum_{n=1}^{\infty} \alpha a_n$ converges to αA .
 - (b) Prove that $\sum_{n=1}^{\infty} (a_n + b_n)$ converges to $A + B$.
 6. Let (a_n) be a sequence of real numbers.
Prove that $a_n \rightarrow 0$ if and only if $|a_n| \rightarrow 0$.
 7. Determine whether the following series converge or diverge. Explain why.
 - (a) $\sum_{n=1}^{\infty} \frac{2n}{3n+1}$
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$$(b) \sum_{n=1}^{\infty} \frac{1}{n^3 + n^2 + n + 1}$$

$$(c) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 + 1}$$

$$(d) \sum_{n=1}^{\infty} (-1)^n \frac{n^3}{2n^3 + 1}$$

$$(e) \sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$$

8. (a) Prove that if $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

(b) Give an example to show that the contrapositive of part (a) is not true.

The contrapositive is: if $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} |a_n|$ converges.

9. (We used this for evaluating geometric series)

Prove that if $-1 < r < 1$, then $\lim_{n \rightarrow \infty} r^n = 0$.
